International Journal of Theoretical Physics, Vol. 34, No. 10, 1995

Rindler Effect for a Nonuniformly Accelerating Observer

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Received September 30, 1994

Both the Klein–Gordon equation and the Dirac equation are dealt with in the generalized Rindler space-time of a nonuniformly accelerating observer. Making use of a new method and introducing a tortoise-type coordinate transformation, it is proved that there exist an event horizon and thermal radiation depending on time in the space-time. The Hawking–Unruh temperature is proportional to the variable acceleration.

A uniformly accelerating Rindler observer along a straight line in Minkowski space-time receives thermal radiation whose temperature is proportional to the accelerating g although another observer at rest in Minkowski space-time receives nothing, thinking of himself as being in the vacuum (Unruh, 1976). We are interested in what is received by a nonuniformly accelerating observer with a variable acceleration $g(t)$ in the Minkowski vacuum.

1. INTRODUCTION

When the acceleration g is constant, the line element in the local Rindler space-time is given as (Misner *et al.*, 1973)

$$ds^2 = -(1 + gx)^2\, dt^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

where g is the coordinate acceleration. The coordinate temperature of thermal radiation relative to the Rindler observer is

$$T = g/2\pi K_B \tag{2}$$

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0020-7748/95/1000-2049$07.50/0 © 1995 Plenum Publishing Corporation

which is also the proper temperature relative to the observer resting at the origin of the Rindler coordinate system because g is his proper acceleration. Here K_B is Boltzmann's constant. It has been proved that the metric (1) is still a vacuum solution of the Einstein equation when g depends on the time t (Tang, 1989)

$$ds^2 = -[1 + g(t)x]^2\, dt^2 + dx^2 + dy^2 + dz^2 \tag{3}$$

Recently we have developed the method suggested by Damour and Ruffini (1976) for dealing with the Hawking radiation of black holes (Zhao and Dai, 1991, 1992). With this approach it is easy to obtain the temperature of a black hole whose mass depends on time. In this paper, we will investigate, making use of the new method, what will be observed by a generalized Rindler observer whose acceleration $g(t)$ is dependent on time. In Section 2, we show the location of the event horizon in the generalized Rindler space-time (3). In Section 3, we deal with the Klein–Gordon equation and give the temperature and the spectrum of radiation of Klein–Gordon particles by means of a generalized tortoise transformation. Then we calculate the Dirac equation in Section 4. Section 5 is devoted to a discussion.

2. EVENT HORIZON

In the space-time (3), the null hypersurface equation

$$g^{\mu\nu} \frac{\partial f}{\partial x^\mu} \frac{\partial f}{\partial x^\nu} = 0 \tag{4}$$

can be reduced to

$$g^{00}\left(\frac{\partial f}{\partial t}\right)^2 + g^{11}\left(\frac{\partial f}{\partial x}\right)^2 = 0 \tag{5}$$

Because

$$\dot{x} = \frac{\partial x}{\partial t} = -\left(\frac{\partial f}{\partial x}\right)\Big/\left(\frac{\partial f}{\partial t}\right) \tag{6}$$

we have

$$\dot{x}^2 = (1 + gx)^2 \tag{7}$$

or

$$x = -\frac{1}{g}(1 \mp \dot{x}) \tag{8}$$

This is the equation that the null hypersurface in the space-time (3) should satisfy. It is also the necessary condition for an event horizon in the space-time.

3. KLEIN–GORDON EQUATION

In the space-time (3), the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{x^{\mu}}\left((\sqrt{-g}\, g^{\mu\nu}\frac{\partial\Phi}{\partial x^{\nu}}\right) + \mu_0^2\Phi = 0 \tag{9}$$

can be written as

$$-\frac{\partial^2 R}{\partial t^2} + \frac{\dot{g}x}{1 + gx}\frac{\partial R}{\partial t} + (1 + gx)^2\frac{\partial^2 R}{\partial x^2} + g(1 + gx)\frac{\partial R}{\partial x} - (1 + gx)^2(\lambda^2 + \mu_0^2)R = 0 \tag{10}$$

$$\frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} + \lambda^2 S = 0 \tag{11}$$

after the separation of variables $\Phi = R(t, x)S(y, z)$. Here, μ_0 is the mass of a Klein–Gordon particle. λ is a constant. Introducing the generalized tortoise transformation (Zhao and Dai, 1991, 1992)

$$x_* = x + \frac{1}{2\kappa}\ln[x - x_H(t)] \tag{12}$$

$$v_* = t - t_0 \tag{13}$$

We can write equation (10) as

$$\frac{(1 + gx)^2[2\kappa(x - x_H) + 1]^2 - \dot{x}_H^2}{2\kappa(x - x_H)\dot{x}_H}\frac{\partial^2 R}{\partial_{x*}^2} + 2\frac{\partial^2 R}{\partial v_*\partial x_*} - \frac{2\kappa(x - x_H)}{\dot{x}_H}\frac{\partial^2 R}{\partial v_*^2} + \frac{2\kappa x\dot{g}(x - x_H)}{\dot{x}_H(1 + gx)}\frac{\partial R}{\partial v_*}$$
$$+ \left\{\frac{\dot{x}_H^2 - (1 + gx)^2}{(x - x_H)\dot{x}_H} + \frac{\ddot{x}_H}{\dot{x}_H} - \frac{x\dot{g}}{1 + gx} + \frac{g(1 + gx)}{\dot{x}_H}[2\kappa(x - x_H) + 1]\right\}\frac{\partial R}{\partial x_*}$$
$$- \frac{2\kappa(x - x_H)}{\dot{x}_H}[(1 + gx)^2(\lambda^2 + \mu_0^2)]R = 0 \tag{14}$$

where κ is an adjustable parameter. It is constant in the tortoise transformation (12). x_H is the location of the event horizon.

From equation (7), we know that

$$\lim_{x \to x_H} \{(1 + gx)^2[2\kappa(x - x_H) + 1]^2 - \dot{x}_H^2\} = 0 \tag{15}$$

So the limit of the coefficient of the term $\partial^2 R/\partial x_*^2$ when $t \to t_0$ and $x \to x_H$ is

$$A = \lim_{\substack{t \to t_0 \\ x \to x_H(t_0)}} \frac{(1 + gx)^2[2\kappa(x - x_H) + 1]^2 - \dot{x}_H^2}{2\kappa \dot{x}_H (x - x_H)}$$

$$= \frac{(1 + gx)[g + 2\kappa(1 + gx)]}{\kappa \dot{x}_H} \tag{16}$$

Selecting the adjustable parameter as

$$\kappa = \frac{g(1 + gx)}{\dot{x}_H(1 - 2\dot{x}_H)} \tag{17}$$

and making use of equation (7), we get $A = 1$. On the other hand, the limit of the coefficient of the term $\partial R/\partial x_*$ is

$$\lim_{\substack{x \to x_H(t_0) \\ t \to t_0}} \frac{1}{\dot{x}_H} \left(\frac{\dot{x}_H^2 - (1 + gx)^2}{x - x_H} + \frac{1}{1 + gx} \{\ddot{x}_H(1 + gx) - x\dot{x}_H \dot{g} \right.$$

$$\left. + g(1 + gx)^2[2\kappa(x - x_H) + 1]\} \right)$$

$$= \lim_{\substack{x \to x_H(t_0) \\ t \to t_0}} \frac{1}{\dot{x}_H} \left(-2(1 + gx)g + \frac{1}{1 + gx} \left\{ \frac{(\dot{g}x_H + \dot{x}_H g)(1 + gx_H)(1 + gx)}{\dot{x}_H} \right. \right.$$

$$\left. \left. - x\dot{x}_H \dot{g} + g(1 + gx)^2[2\kappa(x - x_H) + 1] \right\} \right)$$

$$= \frac{1}{\dot{x}_H} \{-2(1 + gx_H)g + 2g(1 + gx_H)\} = 0 \tag{18}$$

Here we have made use of equation (7),

$$\dot{x}_H^2 = (1 + gx_H)^2$$

$$\ddot{x}_H^2 = (1 + gx_H)(\dot{g}x_H + g\dot{x}_H)/\dot{x}_H \tag{19}$$

Then equation (14) is reduced to

$$\frac{\partial^2 R}{\partial x_*^2} + 2\,\frac{\partial^2 R}{\partial x_* \partial v_*} = 0 \tag{20}$$

when $x \to x_H$. Its solutions are

$$R_1 = e^{-i\omega v_*}$$
$$R_2 = e^{-i\omega v_* + 2i\omega x_*} \qquad (x > x_H) \tag{21}$$

where ω is the energy of a Klein–Gordon particle.

But what is their physical significance? Making use of another method, namely the "conformal flat method" (Zhao, 1992), we can answer the question easily. By using (12) and (13), we can rewrite the two-dimensional part of the line element (3) as

$$\begin{aligned} dS^2 = \Omega^2\, d\tilde{s}^2 = &\\ &-\left\{(1+gx)^2 - \frac{\dot{x}_H^2}{[2\kappa(x-x_H)+1]^2}\right\} dv_*^2 \\ &+\frac{[2\kappa(x-x_H)]^2}{[2\kappa(x-x_H)+1]^2}\,dx_*^2 + \frac{4\dot{x}_H\kappa(x-x_H)}{[2\kappa(x-x_H)+1]^2}\,dx_*\,dv_* \\ = &\frac{2\kappa\dot{x}_H(x-x_H)}{[2\kappa(x-x_H)+1]^2}\left\{\frac{(1+gx)^2[2\kappa(x-x_H)+1]^2 - \dot{x}_H^2}{2\kappa\dot{x}_H(x-x_H)}\,dv_*^2\right. \\ &\left. + 2\,dx_*\,dv_* + \frac{2\kappa(x-x_H)}{\dot{x}_H}\,dx_*^2\right\} \end{aligned} \tag{22}$$

where

$$\Omega^2 = \frac{2\kappa\dot{x}_H(x-x_H)}{[2\kappa(x-x_H)+1]^2} \tag{23}$$

$$\begin{aligned} d\tilde{S}^2 = &\frac{(1+gx)^2[2\kappa(x-x_H)+1]^2 - \dot{x}_H^2}{2\kappa\dot{x}_H(x-x_H)}\,dv_*^2 \\ &+ 2\,dx_*\,dv_* + \frac{2\kappa(x-x_H)}{\dot{x}_H}\,dx_*^2 \end{aligned} \tag{24}$$

When x goes to x_H, we have

$$d\tilde{S}^2 = -dv_*^2 + 2\,dx_*\,dv_* \tag{25}$$

Here, we have used equation (16). It is easy to see that v_* is a null coordinate, the advanced Eddington coordinate, at the future event horizon $x = x_H$. Therefore, the solutions R_1 and R_2 represent, respectively, the ingoing wave and outgoing wave outside the horizon and near it.

Obviously, the ingoing wave R_1 is analytical at the horizon, but the outgoing one is not. It has a logarithmic singularity at the horizon. Following Damour and Ruffini (1976) we can extend the outgoing wave R_2 by analytical continuation to the inside of the horizon through the lower half complex x-plane

$$(x - x_H) \rightarrow |x - x_H| e^{-i\pi} = (x_H - x) e^{-i\pi} \tag{26}$$

$$\tilde{R}_2 = e^{\pi\omega/\kappa} e^{-i\omega v_* + 2i\omega x_*} \qquad (x < x_H) \tag{27}$$

The relative scattering probability of the outgoing waves produced by the horizon is (Sannan, 1988)

$$P_\omega = \exp(-2\pi\omega/\kappa) \tag{28}$$

It is easy to prove that there exists Hawking–Unruh radiation from the event horizon, whose spectrum and temperature are, respectively,

$$N_\omega = (e^{\omega/K_B T} - 1)^{-1} \tag{29}$$

$$\mathrm{T} = \kappa/2\pi K_B \tag{30}$$

κ is given by equation (17). K_B is Boltzmann's constant.

4. DIRAC EQUATION

The spinor base form of the Dirac equation in curved space-times is (Newman and Penrose, 1962; Chandrasekhar, 1976)

$$\begin{aligned} \sqrt{2}\, \nabla_{a\dot{b}} P^a + i\mu_0 \overline{Q}_{\dot{b}} &= 0 \\ \sqrt{2}\, \nabla_{a\dot{b}} Q^a + i\mu_0 \overline{P}_{\dot{b}} &= 0 \end{aligned} \tag{31}$$

where μ_0 is the mass of the Dirac particle. P^a, Q^a, and $\nabla_{a\dot{b}}$ are, respectively, the 2-component spinors and the covariant spinor differentiation expressed with spinor base components. This can be transformed into four coupled equations

$$\begin{aligned} (D + \epsilon - \rho)F_1 + (\bar{\delta} + \pi - \alpha)F_2 &= i(\mu_0/\sqrt{2})G_1 \\ (\Delta + \mu - \gamma)F_2 + (\delta + \beta - \tau)F_1 &= i(\mu_0/\sqrt{2})G_2 \\ (D + \bar{\epsilon} - \bar{\rho})G_2 - (\delta + \bar{\pi} - \bar{\alpha})G_1 &= i(\mu_0/\sqrt{2})F_2 \\ (\Delta + \bar{\mu} - \bar{\gamma})G_1 - (\bar{\delta} + \bar{\beta} - \bar{\tau})G_2 &= i(\mu_0/\sqrt{2})F_1 \end{aligned} \tag{32}$$

where

$$F_1 = P^0, \qquad F_2 = P^1, \qquad G_1 = \overline{Q}^{\dot{1}}, \qquad G_2 = -\overline{Q}^{\dot{0}}$$

$$D = \partial_{0\dot{0}} = l^\mu \partial_\mu, \qquad \Delta = \partial_{1\dot{1}} = n^\mu \partial_\mu, \qquad \delta = \partial_{0\dot{1}} = m^\mu \partial_\mu,$$

$$
\begin{aligned}
\bar{\delta} &= \partial_{1\dot{0}} = \bar{m}^{\mu}\partial_{\mu} \\
\epsilon &= \frac{1}{2}\,(l_{\mu;\nu}n^{\mu}l^{\nu} - m_{\mu;\nu}\bar{m}^{\mu}l^{\nu}) \\
\alpha &= \frac{1}{2}\,(l_{\mu;\nu}n^{\mu}\bar{m}^{\nu} - m_{\mu;\nu}\bar{m}^{\mu}\bar{m}^{\nu}) \\
\gamma &= \frac{1}{2}\,(l_{\mu;\nu}n^{\mu}n^{\nu} - m_{\mu;\nu}\bar{m}^{\mu}n^{\nu}) \\
\beta &= \frac{1}{2}\,(l_{\mu;\nu}n^{\mu}m^{\nu} - m_{\mu;\nu}\bar{m}^{\mu}m^{\nu}) \\
\rho &= l_{\mu;\nu}m^{\mu}\bar{m}^{\nu} \\
\pi &= -n_{\mu;\nu}\bar{m}^{\mu}l^{\nu} \\
\mu &= -n_{\mu;\nu}\bar{m}^{\mu}m^{\nu} \\
\tau &= l_{\mu;\nu}m^{\mu}n^{\nu}
\end{aligned} \tag{33}
$$

ϵ, α, γ, β, ρ, π, μ, τ are the special designations of the spin coefficients defined by Newman and Penrose (1962); l^{μ}, n^{μ}, m^{μ}, and $\bar{m}^{\mu}$ are the null tetrad vectors; they satisfy

$$
\begin{aligned}
l^{\mu}l_{\mu} &= n^{\mu}n_{\mu} = m^{\mu}m_{\mu} = \bar{m}^{\mu}\bar{m}_{\mu} = 0 \\
l^{\mu}n_{\mu} &= -m^{\mu}\bar{m}_{\mu} = 1 \\
l^{\mu}m_{\mu} &= l^{\mu}\bar{m}_{\mu} = n^{\mu}m_{\mu} = n^{\mu}\bar{m}_{\mu} = 0 \\
g_{\mu\nu} &= l_{\mu}n_{\nu} + n_{\mu}l_{\nu} - m_{\mu}\bar{m}_{\nu} - \bar{m}_{\mu}m_{\nu}
\end{aligned} \tag{34}
$$

In the generalized Rindler space-time (3), they are

$$
\begin{aligned}
l_{\mu} &= \frac{1}{\sqrt{2}}\,[(1 + gx), 1, 0, 0] \\
n_{\mu} &= \frac{1}{\sqrt{2}}\,[(1 + gx), -1, 0, 0] \\
m_{\mu} &= \frac{1}{\sqrt{2}}\,[0, 0, 1, i] \\
\bar{m}_{\mu} &= \frac{1}{\sqrt{2}}\,[0, 0, 1, -i]
\end{aligned} \tag{35}
$$

or

$$
\begin{aligned}
l^{\mu} &= \frac{1}{\sqrt{2}}\left[\frac{1}{1+gx}, -1, 0, 0\right] \\
n^{\mu} &= \frac{1}{\sqrt{2}}\left[\frac{1}{1+gx}, 1, 0, 0\right] \\
m^{\mu} &= \frac{1}{\sqrt{2}}[0, 0, -1, -i] \\
\overline{m}_{\mu} &= \frac{1}{\sqrt{2}}[0, 0, -1, i]
\end{aligned} \tag{36}
$$

We calculate the spin coefficients and get

$$
\begin{aligned}
\epsilon &= \gamma = -\frac{1}{2\sqrt{2}}\frac{g}{1+gx} \\
\alpha &= \beta = \rho = \pi = \mu = \tau = 0
\end{aligned} \tag{37}
$$

Then, equation (32) can be reduced to

$$
\begin{aligned}
&\left[\frac{1}{1+gx}\frac{\partial}{\partial t} - \frac{\partial}{\partial x} - \frac{g}{2(1+gx)}\right]F_1 + \left(-\frac{\partial}{\partial y} + i\frac{\partial}{\partial z}\right)F_2 = i\mu_0 G_1 \\
&\left[\frac{1}{1+gx}\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{g}{2(1+gx)}\right]F_2 + \left(-\frac{\partial}{\partial y} - i\frac{\partial}{\partial z}\right)F_1 = i\mu_0 G_2 \\
&\left[\frac{1}{1+gx}\frac{\partial}{\partial t} - \frac{\partial}{\partial x} - \frac{g}{2(1+gx)}\right]G_2 - \left(-\frac{\partial}{\partial y} - i\frac{\partial}{\partial z}\right)G_1 = i\mu_0 F_2 \\
&\left[\frac{1}{1+gx}\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{g}{2(1+gx)}\right]G_1 - \left(-\frac{\partial}{\partial y} + i\frac{\partial}{\partial z}\right)G_2 = i\mu_0 F_1
\end{aligned} \tag{38}
$$

Separating the variables as

$$
\begin{aligned}
F_1 &= R_-(t, x)S_-(y, z) \\
F_2 &= R_+(t, x)S_+(y, z) \\
G_1 &= R_+(t, x)S_-(y, z) \\
G_2 &= R_-(t, x)S_+(y, z)
\end{aligned} \tag{39}
$$

and substituting them into equations (38), we have

$$(1 + gx)^2 \frac{\partial^2 R_\mp}{\partial x^2} - \frac{\partial^2 R_\mp}{\partial t^2} + \frac{x\dot{g} \pm g(1 + gx)}{1 + gx} \frac{\partial R_\mp}{\partial t} + g(1 + gx) \frac{\partial R_\mp}{\partial x}$$
$$+ \left[\pm \frac{\dot{g}}{2(1 + gx)} - \frac{g^2}{4} - (1 + gx)^2(\lambda^2 + \mu_0^2) \right] R_\mp = 0 \qquad (40)$$

$$\frac{\partial^2 S_\mp}{\partial y^2} + \frac{\partial^2 S_\mp}{\partial z^2} + \lambda^2 S_\mp = 0 \qquad (41)$$

Making use of the tortoise coordinates (12) and (13), we can reduce (40) to

$$\frac{\partial^2 R_\mp}{\partial x_*^2} + 2 \frac{\partial^2 R_\mp}{\partial x_* \partial v_*} \mp g \frac{\partial R_\mp}{\partial x_*} = 0 \qquad (42)$$

when $x \to x_H$. Here, equations (7) and (17) have been used.

It is easy to get the solutions of equations (41) and (42) outside the event horizon and near it. They are

$$\begin{aligned} R_\mp^{\text{in}} &= e^{-i\omega v_*} \\ R_\mp^{\text{out}} &= e^{-i\omega v_* + 2i\omega x_*} e^{\pm g x_*} \qquad (x > x_H) \\ S_\mp^{\text{in}} &= S_\mp^{\text{out}} = e^{i(P_y y + P_x z)} \end{aligned} \qquad (43)$$

Extending $R_\mp^{\text{out}}$ by analytical continuation to the region $x < x_H$, through the lower half complex x-plane, we have

$$\tilde{R}_\mp^{\text{out}} = e^{\pi\omega/\kappa} e^{\mp ig\pi/2\kappa} e^{-i\omega v_* + 2i\omega x_*} e^{\pm g x_*} \qquad (x < x_H) \qquad (44)$$

Because the wave functions must be bounded near the event horizon, we get

$$\begin{cases} R_-^{\text{out}} = 0 \\ R_+^{\text{out}} = e^{-i\omega v_* + 2i\omega x_*} e^{-g x_*} \end{cases} \qquad (g < 0,\ x > x_H) \qquad (45)$$

$$\begin{cases} \tilde{R}_-^{\text{out}} = 0 \\ \tilde{R}_+^{\text{out}} = e^{\pi\omega/\kappa} e^{ig\pi/2\kappa} e^{-i\omega v_* + 2i\omega x_*} e^{-g x_*} \end{cases} \qquad (g < 0,\ x < x_H) \qquad (46)$$

$$\begin{cases} R_-^{\text{out}} = e^{-i\omega v_* + 2i\omega x_*} e^{g x_*} \\ R_+^{\text{out}} = 0 \end{cases} \qquad (g > 0,\ x > x_H) \qquad (47)$$

$$\begin{cases} \tilde{R}_{-}^{\text{out}} = e^{\pi\omega/\kappa} e^{-ig\pi/2\kappa} e^{-i\omega v_* + 2i\omega x_* e^{gx_*}} \\ \tilde{R}_{+}^{\text{out}} = 0 \end{cases} \qquad (g > 0, x < x_H) \tag{48}$$

The relative scattering probabilities produced by the horizon are the same,

$$\left|\frac{R_{+}^{\text{out}}}{\tilde{R}_{+}^{out}}\right|^2 = e^{-2\pi\omega/\kappa} \qquad (g < 0) \tag{49}$$

$$\left|\frac{R_{-}^{\text{out}}}{\tilde{R}_{-}^{\text{out}}}\right|^2 = e^{-2\pi\omega/\kappa} \qquad (g > 0) \tag{50}$$

Then we obtain the spectrum of Hawking–Unruh radiation of the Dirac particles from the Rindler event horizon (Sannan, 1988). It is

$$N_\omega = (e^{\omega/K_B T} + 1)^{-1} \tag{51}$$

where $T = \kappa/2\pi K_B$, with κ given by (17).

5. DISCUSSION

Making use of a new method obtained by developing the Damour–Ruffini approach, we have introduced generalized tortoise coordinates and treated the Klein–Gordon equation and the Dirac equation in the generalized Rindler space-time. We get both the location and the Hawking–Unruh temperature of the event horizon with respect to a generalized Rindler observer moving with a nonuniform acceleration. We also get the radiative spectra of the Klein–Gordon particles and the Dirac particles to the observer.

In fact, the most convenient approach to calculating the temperature and the location of the event horizon is the "conformal flat method" (Zhao, 1992). Requiring that the coefficient of dv_*^2 in equation (24) goes to 1 when $x \to x_H$, we can easily get both the location and the temperature of the event horizon shown in (8) and (17).

Equations (8) and (17) give us two sets of solutions about the location and the temperature. They are

$$x_H = -\frac{1}{g}(1 - \dot{x}_H), \qquad \kappa = g(t)/(1 - 2\dot{x}_H), \qquad T = \kappa/2\pi K_B \tag{52}$$

$$x_H = -\frac{1}{g}(1 + \dot{x}_H), \qquad \kappa = -g(t)/(1 - 2\dot{x}_H), \qquad T = \kappa/2\pi K_B \tag{53}$$

The first set is valid when the Rindler observer is accelerating, i.e., $g(t) > 0$. The second set is valid when the observer is decelerating, i.e., $g(t) < 0$.

We see that both the event horizon and the temperature depend on the variable acceleration g.

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